1. Find $\frac{dy}{dx}$
   a) $y = 3x^4 + 8x^3 - 1$
   $$y' = 12x^3 + 24x^2$$
   b) $y = x^6 \tan x$
   $$y' = x^6 \sec^2 x + 6x^5 \tan x$$
   c) $y = \frac{7x + 6}{8x - 6}$
   $$y' = \frac{(8x-6)x - 8(7x+6)}{(8x-6)^2} = \frac{-90}{(8x-6)^2}$$
   d) $y = \frac{\sqrt{x} - 4}{\sqrt{x} + 4}$
   $$y' = \frac{(\sqrt{x}+4)(\frac{1}{2}x^{-\frac{1}{2}}) - (\sqrt{x}+4)^2 x^{-\frac{1}{2}}}{(\sqrt{x}+4)^2}$$
   e) $y = -\frac{6}{x^3} + \frac{6}{\sqrt{x}} - \frac{3}{x}$
   $$y' = -6x^{-2} + 6x^{-\frac{3}{2}} - \frac{3}{x^2}$$
   f) $y = \frac{\sin x}{10x}$
   $$y' = \frac{10x \cos x - 10 \sin x}{100x^2}$$
   g) $y = x^5 + \csc x - \cot x$
   $$y' = 5x^4 - \csc x \cot x + \csc^2 x$$
   h) $y = 6e^x + 2 \sin x \cos x$
   $$y' = 6e^x + 2 \sin x (-\sin x) + 2 \cos x \cos x$$

2. If $f(x) = 7x \sin x$, find $f''(x)$.
   $$f'(x) = 7x \cos x + 7 \sin x$$
   $$f''(x) = 7 \sin x + 7 \cos x$$

3. The position of a particle moving along the x-axis is given by the function $s(t) = 6 \sin t - \cos t$.
   a) Find the particle’s velocity at time $t = \frac{\pi}{4}$.
   $$\mathbf{v}(t) = 6 \cos t + \sin t$$
   $$\mathbf{v}(\frac{\pi}{4}) = 6 \cos \frac{\pi}{4} + \sin \frac{\pi}{4} = \sqrt{2}$$
   b) Find the particle’s acceleration at time $t = \frac{\pi}{4}$.
   $$\mathbf{a}(t) = -6 \sin t + \cos t$$
   $$\mathbf{a}(\frac{\pi}{4}) = -6 \sin \frac{\pi}{4} + \cos \frac{\pi}{4} = -\frac{3\sqrt{2}}{2}$$

4. Find the equation of the tangent and normal lines to the equation $f(x) = -9x^2 + 9x$ where $x = 2$.
   a. Tangent: $y + 18 = -\frac{1}{2} (x-2)$
   b. Normal: $y + 18 = 2(x-2)$

5. A particle moves along a line so that its position at any time, $t \geq 0$, is given by the function $s(t) = 2t^3 - 18t^2 + 30t + 5$ where $s$ is measured in feet and $t$ is measured in seconds.
   a. Find the average velocity on the interval [1, 3].
   b. Find the instantaneous velocity at $t = 2$.
   c. When is the particle at rest?
   d. Find a formula for the acceleration at any time $t$.

   C. when $\mathbf{v}(t) = 0$
   $$0 = 6t^2 - 36t + 30$$
   $$0 = (6t-6)(t-5)$$
   $$t = 1 \text{ second}$$
   $$t = 5 \text{ seconds}$$

   D. $\mathbf{a}(t) = 12t - 36$
7. The following table shows the position of a particle, $S$, at several times, $t$, as the particle moves along a straight line, where $t$ is measured in seconds and $S$ is measured in meters. Find the best estimate possible for the velocity of the particle at $t = 5$ seconds. Indicate units of measure.

<table>
<thead>
<tr>
<th>$t$</th>
<th>3.2</th>
<th>4.0</th>
<th>6.0</th>
<th>7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(t)$</td>
<td>5.7</td>
<td>8.1</td>
<td>7.5</td>
<td>11.2</td>
</tr>
</tbody>
</table>

\[
\frac{S(6) - S(4)}{6 - 4} = \frac{7.5 - 8.1}{2} = -0.3 \text{ m/s}
\]

\[
\frac{S(6) - S(4)}{6 - 4} = \frac{7.5 - 8.1}{2} = -0.3 \text{ m/s}
\]

8. A particle moves along a line so that its position at any time, $t \geq 0$, is given by the function $s(t) = t^3 - 3t^2 + 3t + 1$ where $s$ is measured in feet and $t$ is measured in seconds. Answer each question with appropriate units.

a) Find the displacement during the first 3 seconds.
\[
s(3) - s(0) = 27 - 1 = 26 \text{ ft} - 1
\]

b) Find the average velocity during the first 3 seconds.
\[
\frac{s(3) - s(0)}{3 - 0} = \frac{26}{3} \text{ ft/s}
\]

c) Find the instantaneous velocity when $t = 3$ seconds.
\[
s'(3) = 3t^2 - 6t + 3
\]
\[
s'(3) = 48 \text{ ft/s}
\]

d) Find the acceleration of the particle at $t = 3$ seconds.
\[
s''(t) = 6t - 6
\]
\[
s''(3) = 18 - 6 = 12 \text{ ft/s}^2
\]

e) Find the speed of the particle when $t = 2$ seconds.
\[
|s'(2)| = 12 \text{ ft/s} = 27 \text{ ft/s}
\]
f) At what time(s) is the particle at rest?
\[
v(t) = 0 \quad 3t^2 - 6t + 3 = 0
\]
\[
3(t^2 - 2t + 1) = 0
\]
\[
3(t + 1)(t - 1) = 0
\]
\[
t = -1, 1
\]

9. (Calculator) The position of a particle at time $t$ seconds, $t \geq 0$, is given by $s(t) = t^4 - 3t^2 + t - 3$, where $t$ is measured in seconds and $s(t)$ is measured in meters. Find the particle’s velocity when the acceleration is 0.
\[
v(t) = 4t^3 - 6t + 1
\]
\[
\frac{dv}{dt} = 12t^2 - 6
\]
\[
\frac{1}{2} = t^2
\]
\[
t = \frac{1}{\sqrt{2}}
\]
\[
v(\frac{1}{\sqrt{2}}) = -1.828
\]

10. If $f(x) = \cos x$, find $f^{(206)}(x)$
\[
f'(x) = -\sin x
\]
\[
f''(x) = -\cos x
\]
\[
f'''(x) = \sin x
\]
\[
f^{(4)}(x) = \cos x
\]

11. \[
\lim_{h \to 0} \frac{\tan \left( \frac{\pi}{4} + h \right) - \tan \left( \frac{\pi}{4} \right)}{h} =
\]

\[
definition \ of \ derivative
\rightarrow \ find \ \frac{d}{dx} \tan x \ at \ x = \frac{\pi}{4}
\]

\[
f' = \sec^2 x
\]
\[
f' \left( \frac{\pi}{4} \right) = \frac{1}{\cos^2 \left( \frac{\pi}{4} \right)} = \frac{1}{\left( \frac{\sqrt{2}}{2} \right)^2} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}
\]
12. The graph to the right shows the velocity of a particle moving along the y-axis over a 9 second interval. Use it to answer the following questions for the interval $0 < x < 9$ and justify each response.

a) When is the particle speeding up?

- $(0, 2)$, $(4, 5)$, $(7, 9)$

**When velocity and acceleration have same sign**

b) When is the acceleration of the particle positive?

- $(3, 5)$, $(7, 9)$ when slope is positive

(c) When is the particle moving at a constant speed?

- $(2, 3)$, $(4, 6)$

(d) What is the particle’s speed at time $t = 3$?

- $1 - 2 = -1$

(e) When does the particle change direction?

- $x = 4$

(f) When is the particle moving up?

- $(4, 9)$

13. Use the table to find the following derivatives

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>$\frac{\pi}{2}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>-3</td>
<td>$\frac{1}{4}$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

a) $\frac{d}{dx}[3f(x)]$ at $x = 3$

- $3f'(3) = 3 \left(\frac{\pi}{2}\right) = \frac{3\pi}{2}$

b) $\frac{d}{dx}[f(x)g(x)]$ at $x = 4$

- $f'(x)g(x) + g(x)f'(x) = 4 \cdot \frac{3}{4} = 3\frac{1}{4}$

- $g(4)f(4)f'(4)g'(4) = \frac{(3)(\frac{\pi}{2}) + \frac{3\pi}{2}}{4}$

- $\frac{d}{dx}\left[\frac{1}{4}\right]$ at $x = 4$

- $\frac{5\pi}{4}$

- $\frac{21}{4} \cdot \frac{3}{12} = \frac{7}{4}$