APPLICATIONS OF DIFFERENTIATION
UNIT TEST REVIEW

1. Locate the absolute extrema of the function on the indicated interval.
   \[ f(x) = 3\cos(x) \text{ over } \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \]
   Absolute Minimum: \(-3\)  
   Absolute Maximum: \(0\)
   \[ f'(x) = -3\sin(x) \Rightarrow \sin x = 0 \Rightarrow x = \pi \]

2. Suppose \(f''(x) = 4x^3 - 2x\) and the critical values of the function \(f'(x)\) are \(-1, 0\) and \(1\). Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.
   \[ f''(-1) = -2 \quad f''(1) = 2 \]
   rel \(\max\) at \(x = -1\)  
   rel \(\min\) at \(x = 1\)  
   Test fails at \(x = 0\)

3. Determine whether the Mean Value Theorem can be applied to \(f\) on the closed interval \([a, b]\).
   - If the Mean Value Theorem can be applied, find all values of \(c\) that would satisfy the theorem.
   - If not, explain why not and give an interval where it could be applied.

   (a) \(f(x) = \frac{1}{2}x + \cos x\) on \([0, 2\pi]\)
   \[ f'(c) = \frac{1}{2} - \sin c = \frac{\pi}{2\pi} \]
   \[ \sin c = \frac{1}{2} \]
   \[ c = \frac{\pi}{6}, \frac{5\pi}{6} \]
   (b) \(y = \frac{x^2}{x-3}\) on \([1, 4]\)
   Cannot apply M.V.T.
   Since \(x = 3\) is infinite discontinuity

4. Find the open intervals on which the function is increasing or decreasing. Locate all relative extrema \(f(x) = x^3 - 3x^2 - 24x + 8\)

   Increasing: \((-\infty, -2) U (4, \infty)\)  
   Decreasing: \((-2, 4)\)
   Minimum point: \((4, -72)\)  
   Maximum point: \((-2, 36)\)

5. Find the open intervals on which the function is increasing, decreasing, concave up, and concave down. Identify all relative extrema and point(s) of inflection \(f(x) = x^3 - 8x\)

   Increasing: \((-\infty, -\sqrt{8/3}) U (\sqrt{8/3}, \infty)\)  
   Decreasing: \((-\sqrt{8/3}, \sqrt{8/3})\)
   Concave Up: \((0, \infty)\)  
   Concave Down: \((-\infty, 0)\)
   Relative Minimum: \(\approx -8.709 @ x \approx 1.633\)  
   Relative Maximum: \(\approx 8.709 @ x \approx -1.633\)
   Point(s) of Inflection: \((0, 0)\)
6. Determine whether Rolle's Theorem can be applied to $f$ on the indicated interval.
   - If Rolle's Theorem can be applied, find all values of $c$ on the interval that would satisfy Rolle's Theorem.
   - If it cannot, explain why not and give an interval where Rolle's Theorem could be applied.

   (a) $f(x) = 3x^2 - 12x + 11$ on $[0, 4]$
   \[
   f'(x) = 6x - 12 = 0 \\
   x = 2 \\
   c = 2 \text{ satisfies Rolle's Theorem on } (0, 4)
   \]

   (b) $g(x) = 2x^2 - 7$ on $[0, 3]$
   \[
   g(0) = -7 \\
   g(3) = 11 \\
   \text{Rolle's does not apply } g(0) \neq g(3)
   \]

7. A cylindrical can holds 1 liter (1000 $cm^3$) of liquid. How should the height and radius be chosen to minimize the amount of material needed to manufacture the can?

   \[
   V = \pi r^2 h \\
   h = \frac{1000}{\pi r^2} \\
   A = 2\pi r^2 + 2\pi rh \\
   A(r) = 2\pi r^2 + \frac{2000}{r} \\
   \frac{d}{dr}A(r) = 4\pi r - \frac{2000}{r^2} \\
   4\pi r^3 = 2000 \\
   r = \sqrt[3]{\frac{500}{\pi}} \approx 5.419 cm
   \]

   \[
   h = \frac{1000}{\frac{500}{r^2}} \approx 10.839 cm
   \]

8. A poster is to have an area of 180 square inches with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

   \[
   \text{Print Area} = (x-2)(y-3) \\
   \text{P.A.} = \left(\frac{180}{y} - 2\right) - 3 \\
   2y^2 = 540 \\
   y^2 = 270 \text{ in} \\
   y = 3\sqrt{30} \text{ in} \\
   P = 2\sqrt{30} \text{ in} \\
   x = \frac{180}{y} - 2y - \frac{540}{y} \\
   x = \frac{3\sqrt{30}}{y} - 2y - \frac{540}{y} \\
   x = \frac{540}{y^2} \\
   \]