Approximating area using Riemann sums

1. a) Approximate the area under the graph of \( f(x) = \frac{1}{x} \) from \( x = 1 \) to \( x = 5 \) using the right endpoints of four subintervals of equal length. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   b) Repeat part a) using left endpoints.

2. Approximate the area under the graph of \( f(x) = 25 - x^2 \) from \( x = 0 \) to \( x = 5 \) using the midpoints of five subintervals of equal length. Sketch the graph and the rectangles.

3. a) Approximate the area under the graph of \( f(x) = x^2 + 1 \) from \( x = -1 \) to \( x = 2 \) using the right endpoints of three subintervals of equal length. Sketch the graph and the rectangles.
   b) Improve your estimate by using six subintervals.
   c) Repeat parts a) and b) using left endpoints.
   d) Repeat parts a) and b) using midpoints.
   e) From your sketches in parts a), c), and d), which appears to be the best estimate?

4. a) Approximate the area under the graph of the function shown to the right from \( x = -2 \) to \( x = 3 \) using the right endpoints of five subintervals of equal length.
   b) Repeat part a) using left endpoints.
   c) Repeat part a) using midpoints.

5. | \( x \)  | -5  | -3  | 0  | 1  | 5  |
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<tbody>
<tr>
<td>( f(x) )</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>11</td>
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   a) Given the values for \( f(x) \) on the table above, approximate the area under the graph of \( f(x) \) from \( x = -5 \) to \( x = 5 \) using the left endpoints of four subintervals.
   b) Repeat part a) using right endpoints.
   c) Could you do this problem using midpoints of four subintervals? Explain.

6. Left, midpoint, and right Riemann sums were used to estimate the area between the graph of \( f(x) \) and the \( x \)-axis on the interval \([3, 7]\). We know that \( f \) is a function such that \( f(x) > 0 \) and \( f'(x) < 0 \) on \([3, 7]\). The same number of subintervals were used to produce each approximation. The estimates were 1.345, 1.578, and 1.723.

Which rule produced each estimate? Justify your answer.
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1. a) \( A \approx 1.283 \). Underestimate.

   ![Graph showing underestimation](image)

   b) \( A \approx 2.083 \). Overestimate.

   ![Graph showing overestimation](image)

2. \( A \approx 83.75 \)

   ![Graph showing area estimation](image)

3. a) \( A \approx 8 \)

   ![Graph showing area estimation](image)

   b) \( A \approx 6.875 \)

   ![Graph showing area estimation](image)
4.  
   a)  \( A \approx 11.7 \)  
   b)  \( A \approx 10 \)  
   c)  \( A \approx 11.1 \) 

5.  
   a)  Left endpoints: \( A \approx 78 \)  
   b)  Right endpoints: \( A \approx 81 \)  
   c)  No. We do not know the values for \( f(x) \) at the midpoints of each interval. 

6. Since \( f'(x) < 0 \) on \([3, 7]\), the function \( f(x) \) decreases on \([3, 7]\). This means that in each subinterval the values of the function for the left endpoint are higher than both the values for the right point and the values for the midpoint of the subinterval. It also means that the values of the function for the right endpoint of each subinterval are the lowest for that subinterval. So the estimates were 1.345 for the right Riemann sum, 1.578 for the midpoint sum, and 1.723 for the left Riemann sum.

7. 