Important Topics:
- Differentiability and Continuity
- Derivative and Tangent Line
- Local Linearity
- Rules of Differentiation (power rule, product rule, quotient rule, chain rule, trig derivatives, other transcendental function rules: $e^x$, $\ln x$, $a^x$
- Inverse functions and derivative properties
- Symbolic Differentiation

True or False

1. The slope of the tangent line to the differentiable function $f$ at the point $(3, f(3))$ is $f'(3 + h) - f(3)$

2. If a function has derivatives from both the left and the right at a point, then it is differentiable at that point.

3. If a function is differentiable at a point, then it is continuous at that point.

4. If a function is continuous at a point, then it is differentiable at that point.

5. If $f'(c)$ and $g'(c)$ are zero and $h(x) = f(x)g(x)$, then $h'(c) = 0$

6. The second derivative is the rate of change of the first derivative.

7. When the velocity of an object is constant, then its acceleration is zero.

8. If $f$ and $g$ are differentiable functions of $x$ and $h(x) = f(g(x))$, then $h'(x) = f'(g(x))g'(x)$.

9. The domain of $y = \sin^{-1} x$ is $-1 \leq x \leq 1$.

10. The derivative of $y = e^{2x}$ is $2(\ln 2)e^{2x}$

11. The speed of a particle at $t = c$ is given by the value of the velocity at $t = c$.

12. If $y$ is a differentiable function of $u$, $u$ is a differentiable function of $v$, and $v$ is a differentiable function of $x$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$

13. If $y = \sqrt{1-x}$, then $y' = \frac{1}{2}(1-x)^{-\frac{1}{2}}$

14. It is possible to estimate the value of a function $f$ at $x = a$ using $y = f'(a)(x-a) + f(a)$

15. The graph of $f$ is given at right. State the $x$-values where the function is not differentiable.
16. If \( f(x) = e^x \), which of the following is equal to \( f'(e) \)?

A. \( \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \)  
B. \( \lim_{h \to 0} \frac{e^{x+h}}{h} \)  
C. \( \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \)  
D. \( \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \)

17. \( \frac{d}{dx} \left( \frac{1}{x^3} - \frac{1}{x} + x^2 \right) \) at \( x = -1 \) is

A. 6  
B. 2  
C. 0  
D. -4

18. Differentiate \( y = \frac{3}{4+x^2} \)

19. If \( f \) is a function where \( \lim_{x \to 3} \frac{f(x) - f(3)}{x-3} = 0 \), which of the following must be true?

A.) \( f(3) = 0 \)  
B.) The limit of \( f(x) \) as \( x \) approaches 3 does not exist.  
C.) The derivative of \( f \) at \( x = 3 \) is 0.  
D.) \( f \) is not defined at \( x = 3 \).

20. \( \lim_{h \to 0} \frac{\tan(4(x+h)) - \tan(4x)}{h} \)

A.) 0  
B.) \( 4 \sec^2(4x) \)  
C.) \( \sec^2(4x) \)  
D.) \( 4 \cot(4x) \)

21. \( \frac{d}{dx} (\ln e^{5x}) = \)

A.) \( \frac{1}{e^{5x}} \)  
B.) \( \frac{5}{e^{5x}} \)  
C.) 5x  
D.) 5
22. Let \( f(x) = \frac{1}{4}x^3 + x - 1 \) and let \( g(x) = f^{-1}(x) \) the inverse of \( f(x) \). Then, \( g'(3) = \)

A.) \( \frac{4}{31} \) B.) \( \frac{1}{6} \)
C.) \( \frac{1}{4} \) D.) \( \frac{1}{3} \)

23. At \( x = 3 \), the function \( f(x) = \begin{cases} x^2 & , x < 3 \\ 6x - 9 & , x \geq 3 \end{cases} \) is

A.) undefined B.) continuous but not differentiable.
C.) differentiable but not continuous.
D.) both continuous and differentiable.

24. If \( H(x) = \sqrt{g(x)} \) and \( g(3) = 10 \) and \( g'(3) = 4 \). What is the value of \( H'(3) \)?

A.) \( \frac{1}{4} \) B.) \( \frac{1}{2\sqrt{10}} \)
C.) \( \frac{2}{\sqrt{10}} \) D.) 2

25. At what point on the graph of \( y = \frac{1}{2}x^2 \) is the tangent line parallel to the line \( 2x - 4y = 3 \)

A) (2, 2) B) \( \left( \frac{1}{2}, -\frac{1}{2} \right) \)
C) \( \left( \frac{1}{2}, \frac{1}{8} \right) \) D) \( \left( 1, -\frac{1}{4} \right) \)

26. Find the derivative of \( x^2 f(x) \)

A) \( 2x f'(x) \) B) \( x[f(x) + 2f'(x)] \)
C) \( x^2 f'(x) \) D) \( x[f'(x) + 2f(x)] \)

27. A particle moves along the x-axis so that at any time \( t \) its position is \( x(t) = \frac{1}{2} \sin t + \cos(2t) \). What is the second derivative (acceleration) of the particle at \( t = \frac{\pi}{2} \)?

A. 0 B. \( \frac{7}{2} \) C. \( \frac{5}{2} \) D. \( \frac{3}{2} \)
28. For what point of the graph of \( y = x e^{-2x} \) is the tangent line horizontal?

A. \((-1, -e^2)\)  
B. \((-\frac{1}{2}, -\frac{e^2}{2})\)  
C. \((\frac{1}{2}, 0)\)  
D. \((\frac{1}{2}, \frac{1}{2e})\)

29. Given that \( f(-3) = 4 \) and \( f'(-3) = 2 \), which of the following is the tangent line approximation of \( f(-3.1) \)?

A. 3.8  
B. 3.9  
C. 4.018  
D. 4.1

30. Shown below is a table of values from two different differentiable functions \( f \) and \( g \). Use the table to find the value of each expression below. Show work for full credit.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>2</td>
<td>-3</td>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

A.) \( A(x) = f(x) - 3g(x) \) Find \( A'(2) \)  
B.) \( B'(3) \) if \( B(x) = 3f \cdot g \)

C.) \( D'(1) \) if \( D(x) = \frac{1}{g(x)} \)  
D.) \( H(x) = (g(f(x^2))) \) find \( H'(1) \)

E.) \( P'(3) \) if \( P(x) = \sin(g(x)) \)  
F.) \( R(x) = \sqrt{f(x^2)} \) find \( P'(0) \)

G.) Find the equation of the tangent line of \( f(x) \) at \( x = 3 \)  
H.) Find the tangent line approximation of \( f(2.9) \) given \( f(3) = 6 \)
## UNIT 2 REVIEW KEY

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<tr>
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<tbody>
<tr>
<td>2. F</td>
<td>10. F</td>
<td>18. $y' = \frac{-6x}{(4+x^2)^2}$</td>
<td>26. D</td>
<td>30E. $-1$</td>
</tr>
<tr>
<td>7. T</td>
<td>15. ${-3, 0, 2, 4, 5}$</td>
<td>23. D</td>
<td>30B. $-18$</td>
<td></td>
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<tr>
<td>8. T</td>
<td>16. D</td>
<td>24. C</td>
<td>30C. $\frac{3}{4}$</td>
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</table>